USPT Problem 11

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Problem Statement

A washer on a vertical steel rod may start spinning instead of simply sliding down. Study the motion of the washer, the sliding-spinning transition, and determine the terminal velocity.

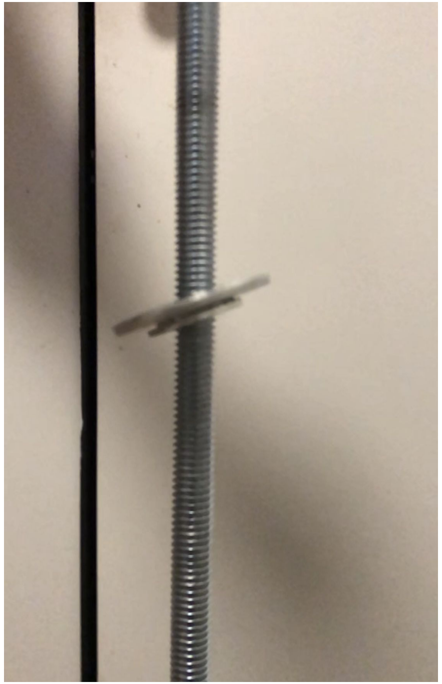
Approach

- Preliminary experiments to confirm simplifying assumptions.
- Make a simplified model for the terminal condition
- Use simple mathematics to predict the behavior of the system
- Discuss changes to the set up that can improve accuracy in the future

Initial Experiment to Confirm a Simplifying Assumption

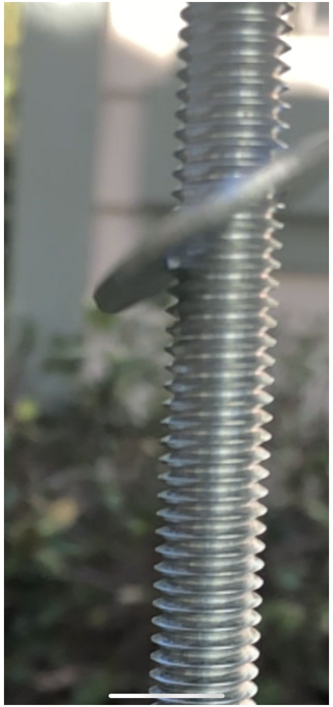
- The rod is threaded: We did the experiment with a threadless rod and it did not work as expected, moreover the rod in the video is threaded too.

- The washer sticks to the rod: We assumed that both ends of the washer stick to the threads of the rod and the tilt of the washer is only dependent on the inner radius.

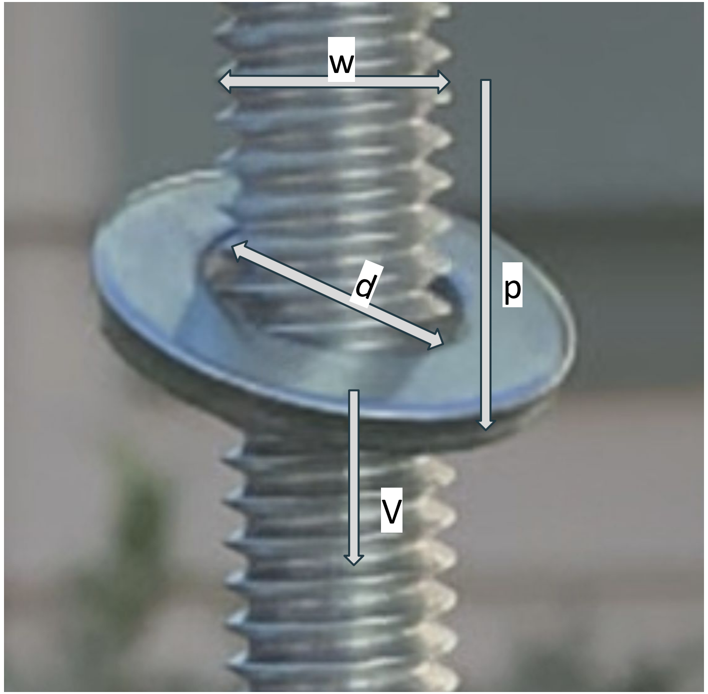


Close Up Images

The washer locks onto the rod at two contact points, following the rings on the threaded rod.



Theoretical Model



d - Inner *Diameter of Washer*

w - *Width of Rod*

p - *Pitch of Rod (Threads per meter)*

m - *Mass of the Washer*

I - *Moment of Inertia of Washer*

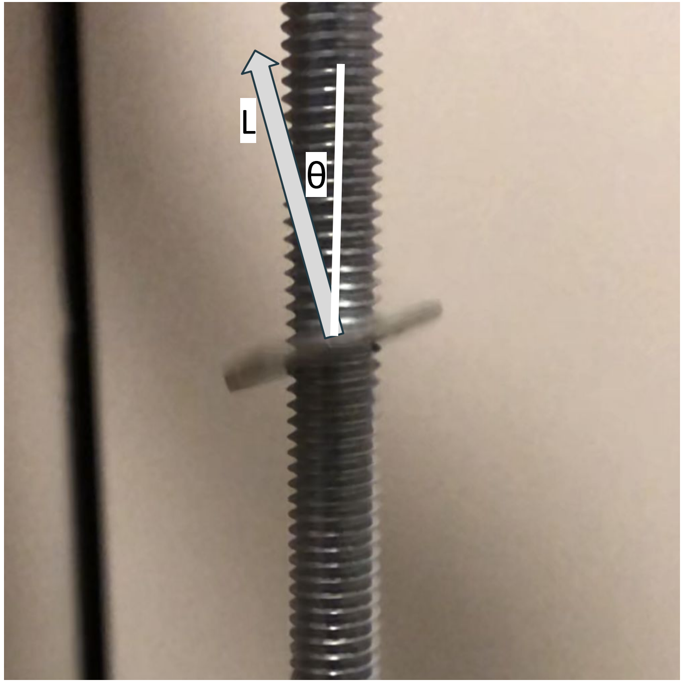
ω - *Terminal Angular Velocity*

v - *Terminal Velocity*

μ - *Coefficient of Sliding Friction (Rod)*

s - *Slope of thread*

R - *Outer Diameter of Washer*

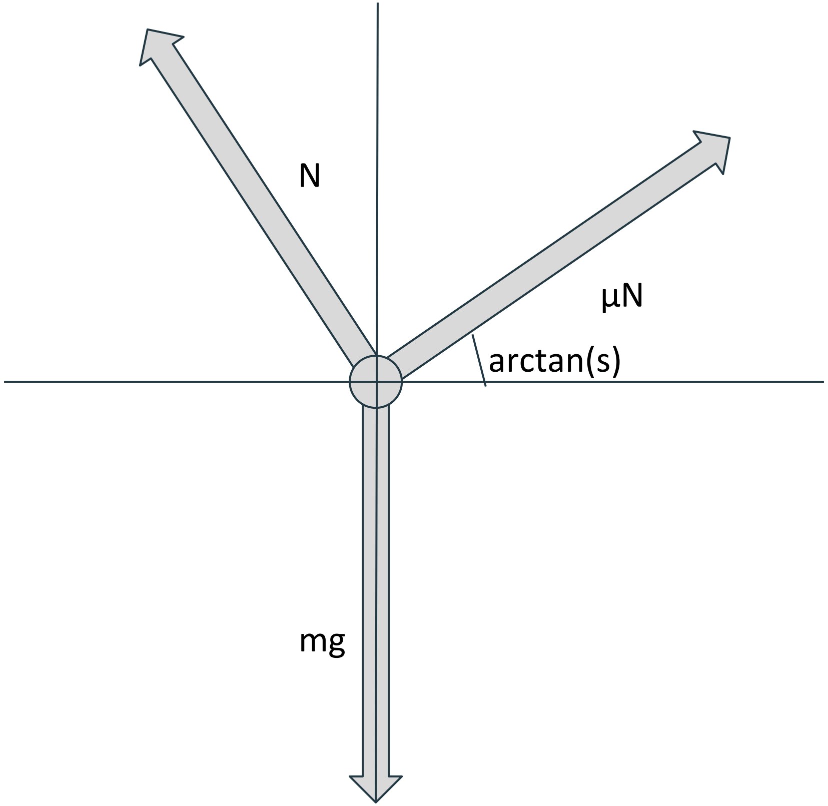


$$\begin{aligned}\mathbf{L} &= L_z\hat{k} + L_y\hat{j} + L_x\hat{i}\\ &= L_z(\hat{k} + \tan(\theta)(\sin(\omega t)\hat{j} + \cos(\omega t)\hat{i}))\end{aligned}$$

$$|\mathbf{L}| = L_z\sqrt{1+\tan^2(\theta)} = I\omega$$

$$\therefore L_z = \frac{I\omega}{\sqrt{1+\tan^2(\theta)}}$$

$$\begin{aligned}|\tau| &= \left|\frac{d\mathbf{L}}{dt}\right|\\ &= \omega L_z \tan(\theta)\end{aligned}$$



$$N\left(\frac{1}{\sqrt{1+s^2}}+\frac{\mu s}{\sqrt{1+s^2}}\right)=mg$$

$$N=\frac{mg\sqrt{1+s^2}}{1+\mu s}$$

And so the torque,

$$
\begin{aligned}
|\tau| &= \frac{w}{2} N \left(\frac{s}{\sqrt{1+s^2}} - \frac{\mu}{\sqrt{1+s^2}} \right) \\
&= \frac{mgw(s-\mu)}{2(1+\mu s)}
\end{aligned}
$$

Finally,

$$\omega L_z \tan(\theta) = \frac{mgw(s-\mu)}{2(1+\mu s)}$$

And thus we have,

$$\omega = \sqrt{\frac{mgw(s-\mu)\sqrt{1+\tan^2(\theta)}}{2I(1+\mu s)\tan(\theta)}}$$

Where:

$$
\begin{aligned}
s &= \frac{1}{2\pi p} \\
\theta &= \arccos\left(\frac{w}{d}\right) \\
I &= \frac{m}{8}(R^2 - d^2)
\end{aligned}
$$

$$\boxed{v = \frac{w}{2}\sqrt{\frac{mgw(s-\mu)\sqrt{1+\tan^2(\theta)}}{2I(1+\mu s)\tan(\theta)}}}$$

Our Experiment

- 5 similar washer
- A threaded rod
- Marker and marked a distance of 39.5cm on the rod

Washer 1

Mass: 3g
Inner-R: 0.54cm
Outer-R: 0.97cm
Thickness: 1.8mm

Distance: 39.5cm
Time Avg: 13.4 seconds
Terminal Velocity: 0.0295 m/s

Predicted Vt: 0.107 m/s

Washer 2

Mass: 4g
Inner-R: 0.50cm
Outer-R: 1.11cm
Thickness: 1.5mm

Distance: 39.5cm
Time Avg: 9.84 seconds
Terminal Velocity: 0.0402 m/s

Predicted Vt: 0.092 m/s

Washer 3

Mass: 5g
Inner-R: 0.59cm
Outer-R: 1.25cm
Thickness: 1.1mm

Distance: 39.5cm
Time Avg: 20.56 seconds
Terminal Velocity: 0.0192 m/s

Predicted Vt: 0.075 m/s

Washer 4

Mass: 7g
Inner-R: 0.55cm
Outer-R: 1.59cm
Thickness: 1.1mm

Distance: 39.5cm
Time Avg: 14.61 seconds
Terminal Velocity: 0.0270 m/s

Predicted Vt: 0.057 m/s

Washer 5

Mass: 19g
Inner-R: 0.67cm
Outer-R: 1.80cm
Thickness: 2.2mm

Distance: 39.5cm
Time Avg: 14.79 seconds
Terminal Velocity: 0.0267 m/s

Predicted Vt: 0.047 m/s

When do our assumptions break down?

- We notice that the washer sometimes slips down the rod even in terminal behavior.
- The thickness might become non negligible.
- We may have to come up with an effective width to compensate for the threads.
- The threads might be uneven and the averaging might not compensate.
- The mass distribution of the washer may be asymmetrical.
- The coefficient of friction might vary along the threads.

How we plan to improve the experiment

- Try to analyze the sound produced by the washer, the spectrogram showed interesting behaviour. Certain frequency has higher intensity and more persistent.
- Try to simulate the problem
- Since we reached the right order of magnitude, it seems like the rest can be chalked up to measuring everything more accurately.

Thank you :)